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Equivalence of the Theories of Reciprocity and General Relativity

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Abstract

Khan's theory of reciprocity has been shown to be equivalent to the theory of general relativity (in a conformally flat space-time) in that the same predictions are made physically. It is proved that, since "centrifugal forces" are used by Khan, gravitational phenomena *are* being considered equal in status to electromagnetic phenomena, and hence the difference claimed to exist between Milne's theory and Khan's theory disappears.

1. *Introduction*

The theory presented by Khan (1968, 1972), which generalizes the principle of special relativity in a natural way, is aesthetically very pleasing. However, the calculations involved are very tedious, and there may be doubts about the general validity of the theory as the velocities are assumed to be infinitely differentiable for all observers. Then again, the generalization to more than one space-like dimension is in itself a very tedious problem.

It has been noticed by Khan that his theory bears a strong resemblance to Milne's theory of kinematic relativity (Milne). However, he claims that there is a fundamental difference between his theory and Milne's theory in that Milne's theory treats gravitation and electromagnetism on the same footing, while Khan's theory treats electromagnetism as more basic.

We shall see that, from the point of view of physical predictions, there is no difference between the theories. Assuming his claim to be correct (that this is the only difference), we shall see that both Milne's theory and Khan's theory are equivalent (in some sense) to the general theory of relativity. There is, in fact, only a difference of the point of view which is taken in

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defining the fundamental physical variables. In another work (Qadir, 1976), the author has dealt with a wider class of theories which give physically equivalent (in some sense) results, but that is beyond the scope of this work.

We conclude that though the theory of reciprocity (Khan, 1968; 1972) is "beautiful" in that it shows reciprocity explicitly, we can *equivalently* (in some sense) consider the theory of general relativity, which is more "useful" in that calculations are much more simple.

2. *Khan's Theory of Reciprocity*

Khan bases his theory on the principle of reciprocity which may be stated as "*all* motions are relative." This would include constant acceleration for example. In using this principle he completely rejects Mach's principle—as opposed to the point of view of Brans and Dicke (1961) or Sciama (1969), who use it rigidly. There are two basic objections made to this point of view in general. The first is that it revives the clock paradox. The second, and more serious objection, is that there is a physical means of measuring zero acceleration. Let us look at the answers to both problems.

Khan's "ideal" clocks are *defined* by the principle of reciprocity, being a clock which respects reciprocity. Thus, when two observers A and B who have moved arbitrarily relative to each other meet again, they must agree on the time elapsed since they last met. This being the definition of the units of measurement of time, there can be no clock paradox. Let us consider only zero or constant accelerations to be allowed, i.e., in every region one of the following two equations holds:

$$a(t) = 0 \tag{2.1}$$

$$a(t) = \text{const} \tag{2.2}$$

Then we require that, since for the region where equation (2.1) holds A sees B's clock as slowed down, in the region where equation (2.2) holds he must see B's clock as speeded up in such a way that the time measurements agree at the end. This leads us to a formula for the apparent speeding up of B's clock, assuming negligible relative speed in the region where equation (2.2) holds:

$$dt'/dt = 1 + xa/c^2 \tag{2.3}$$

where x is the distance from A to B, c the speed of light, and t' is B's time as measured by A. Obviously the same formulas apply for B's measurement of A's time. Notice that this assumes that

$$v/c \ll 1 \tag{2.4}$$

v being the relative speed between A and B, in the region where equation (2.1) holds. Also notice that this is merely a definition of the measurement of time and *not* a prediction that a given physical clock will behave in this manner.

The other problem to be faced by the theory of reciprocity is that the frame of zero acceleration *can* be determined. This may be done by considering an unsupported object. If it accelerates relative to the frame it is a noninertial frame. An inertial frame is one in which unsupported objects appear to "float." The observation, being a purely local one, appears to give evidence contrary to the principle of reciprocity. That such a frame actually exists would be attested by anyone who has been in free fall.

Though not explicitly considered by Khan, the theory of reciprocity obviously incorporates the implicit solution of this problem by the introduction of "fictitious forces." It should be noted that all effects of "force" incorporated into the stress–energy tensor become "fictitious forces" in general relativity, so the introduction of such "forces" is not, in itself, a serious objection. That Khan uses "fictitious forces" has not been explicitly admitted in his work. However, the galactic red-shift derived by him

$$\Delta\lambda/\lambda = GM/cL^2 \tag{2.5}$$

uses a picture with a "centrifugal force". Here we have the following: λ is the wave-length of electromagnetic radiation from a galaxy, $\Delta\lambda$ is the increase of wave-length, G is the gravitational constant, M is the mass of the galaxy, and L is the average radius of the galaxy. This was derived by using Hubble's law (Hubble, 1929)

$$v = H_0 x \tag{2.6}$$

where x is the distance of the galaxy from the earth, H_0 is Hubble's constant, v is the apparent speed of recession of the galaxy, and by using equation (2.2), using the value for a

$$a = GM/L^2 \tag{2.7}$$

This is clearly derived from a picture using "centrifugal force" to give a "centrifugal acceleration."

It should be noticed here that Khan's claim that he does not give a fundamental role to gravitation is not consistent with the calculation of a red-shift due to a gravitational mass [in equation (2.5)], which appears as a GM/L^2 term. Thus the claimed distinction between reciprocity and Milne's theory disappears and the theories are seen to be equivalent. Our claim of equivalence between reciprocity and general relativity would then also extend to kinematic relativity if the distinction claimed by Khan is *in fact the only distinction.*

We see that no new predictions have been made by reciprocity. The measurement of time is different by definition, and the picture of a curved space-time has been replaced by a flat space-time with fictitious forces. Thus there is only a difference of the point of view taken, but not an essential difference of physical predictions. We shall call such theories *physically equivalent theories* (Qadir, 1976), if no difference of physical prediction exists apart from the difference of the definition of measurement. The apparent difference of the prediction of the red-shift is in fact due to an expanding universe in the relativistic sense, but held in check by the "fictitious forces."

3. *Equivalence of the Theories*

The foregoing discussion leads us to expect that the theories of reciprocity and general relativity are equivalent. However, we still need to prove the fact. When we say "equivalent" we mean "physically equivalent" as defined in the previous section. To see this we must understand the principle of reciprocity in a broader context than has so far been presented. Let us therefore examine the more basic "principles" derived from the principle of equivalence.

The first "principle" to be derived is the physical invariance under translation. This is obtained by considering "no motion" in the principle of reciprocity. Either this can be read in the "strong" (global) sense that the universe looks the same under space-time translations, or in the "weak" (local) sense that physical quantities are invariant under space-time translations. Many relativists believe that the global assumption becomes too restrictive [which is not to say that many relativists accept this as the cosmological principle, (e.g., Bondi, 1960)]. This is certainly the spirit of Khan's work. Now consider a finite universe (not allowed by the strong principle) with a definable center. Clearly all physical laws are not invariant under translations literally, since the observer at the center will give space-time isotropy of the universe *in every way* as a physical law, which would not be in keeping with the observations of other observers. Thus in the weak sense it must reduce to the requirement that there be a continuous transformation from one frame to the other frame.

Now consider the second "principle" that can be derived from the principle of reciprocity if we put "uniform motion" in it. Again we are forced to conclude that an infinite number of stars are travelling at all possible speeds relative to any inertial observer, but that only *the same* finite number can be observed from any inertial frame. As this principle is really the principle of special relativity this may seem surprising. However, it may be seen to be required by considering the red-shift of the stars observed by two observers A and B with A moving uniformly relative to B. They must see the same number of red-shifted (and of blue-shifted) stars for the "strong" form of the principle of special relativity to hold. This does not seem possible to arrange. Thus it is easily possible to obtain a frame of reference which, relative to the average motion of the stars, may be taken as the absolute rest-frame, namely, the frame of reference in which all stars (i.e., in all directions) appear equally red-shifted. This does not, however, invalidate the principle of special relativity, which can be read as saying that there exists a continuous transformation from one inertial frame of reference to another.

Similarly, we can consider the third "principle" derivable from the principle of reciprocity that relatively accelerated frames are equivalent. We can easily choose a zero acceleration frame of reference as shown before. However, we notice that we generally use another frame of reference as if it were a zero acceleration frame, namely the frame of reference of the earth. A freely falling object is said to accelerate by gravity, whereas *it* is in the zero acceleration frame previously defined, and we are the perpetually accelerated frame. Thus this principle must also be read in the weak sense that there exists a continuous transformation from one accelerated frame to another.

Thus we see that we have been forced to take the principle of reciprocity in the weaker sense. Thus we may restate it as the weak principle of reciprocity that there exists a transformation between any two frames in relative motion which is continuous if the motion is infinitely differentiable. The equivalence of physical laws in the frames then *defines* the "physical laws." Thus the law of complete symmetry of the universe according to the observer at the center of the universe will *not* be allowed as a physical law. Similarly the law that stars are equally red-shifted in all directions will *not* be allowed as a physical law. Similarly the falling (or nonfalling) of unsupported objects will *not* be allowed as a physical law.

Having so defined our physical laws, and measurement of the basic variables, if two theories make the same predictions physically, up to differences due to a change in the meaning of a physical law or process of measurement, the theories will be called *physically equivalent*. Thus we can state the following theorem:

> *Theorem.* The theory of reciprocity is physically equivalent to the theory of general relativity for conformally flat space-times.

Proof. According to general relativity, for any two conformally flat space-times there exists a conformal transformation which maps the metrics at any point into each other

$$d\tilde{s}^2 = \Omega^2 \, ds^2 \tag{3.1}$$

where Ω^2 is the conformal factor, ds and $d\tilde{s}$ are the metrics in the two space-times, Ω^2 being a continuous scalar field on the manifold. Hence the theorem.

It may be possible to extend the statement of the theorem to conformally curved space-times if we can apply Dicke's units transformations (Dicke, 1962). However, the theory of reciprocity as stated by Khan does not consider problems that would be dealt with in conformally curved space-times (of shear, twist, etc. of neighboring geodesics).

4. *Remarks and Conclusion*

It is the author's point of view that principles such as Mach's principle or the principle of reciprocity should not yield physically different theories in themselves since they specify the procedure of measurement and determine which laws to reject as physical laws. This is not to reduce the importance of Mach's principle, if it bases itself on some physical interactions—as suggested by Sciama (1969), or the principle of reciprocity if *it* bases itself on the "natural" measurement of space-time intervals. However if Mach's principle merely says that a given frame is the most convenient or the principle of reciprocity merely says that a certain definition of the measurement of time is the most pleasing, they could equally well be dropped from the theory. They may be taken as convenient pointers as to how to define variables and what to consider as physical laws but no greater weight should be attached to them.

It is to be noticed that Khan assumes the ability of all observers to meet physically. Thus the general relativistic black holes cannot be allowed in his theory (as stated by him). It may be interesting to see how reciprocity would deal with the problem of black holes and gravitational collapse. It may also be interesting to see if Dicke's units transformations can be applied to make black holes consistent with reciprocity (if they can be applied in that case).

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